MATH-903 Partial Differential Equations-I

Credit hours: 3-0
Prerequisite: None

Course Objectives: Modern technology requires a deeper knowledge of the behavior of real physical phenomena. Mathematical models of real world phenomenon are formulated as algebraic, differential or integral equations (or a combination of them). After the construction of equations the study of their properties is necessary. At this stage the theory of ordinary differential equations plays a significant role. Partial Differential Equations (PDEs) are at the heart of applied mathematics and many other scientific disciplines. PDEs are at the heart of many scientific advances. The behavior of many material object in nature, with time scales ranging from picoseconds to millennia and length scales ranging from sub-atomic to astronomical, can be modelled by PDEs or by equations with similar features. Indeed, many subjects revolve entirely around their underlying PDEs. The role of PDEs within mathematicsand in other sciences is fundamental and is becoming increasingly significant. At the same time, the demands of applications have led to important developments in the analysis of PDEs, which have in turn proved valuable for further different applications. The goal of the course is to provide an understanding of, and methods of solution for, the most important types of partial differential equations that arise in Mathematical Physics. Advanced topics such as week solutions and discontinuous solutions of nonlinear conservation laws are also considered.

Detailed Course Contents: First-order Partial Differential Equations: Linear First-order Equations, The Cauchy Problem for First-order Quasi-linear Equations,Fully-nonlinear First- order Equations, General Solutions of Quasi-linear Equations. Second-order Partial Differential Equations: Classification and Canonical Forms of Equations in Two Independent Variables, Classification of Almost-linear Equations in R^n . One Dimensional Wave Equation: The Wave Equation on the Whole Line. D' Alembert Formula, The Wave Equation on the Half-line, Reflection Method. Mixed Problem for the Wave Equation, Inhomogeneous Wave Equation, and Conservation of the Energy. One Dimensional Diffusion Equation: The Diffusion Equation on the Whole Line, Diffusion on the Half- line, Inhomogeneous Diffusion Equation on the Whole Line, Maximum- minimum Principle for the Diffusion Equation. Weak Solutions, Shock Waves and Conservation Laws: Weak Derivatives and Weak Solutions Conservation Laws, Burgers' Equation, Weak Solutions. Riemann Problem, Discontinuous Solutions of Conservation Laws, Rankine-Hugoniot Condition.
The Laplace Equation: Harmonic Functions. Maximum-minimum Principle, Green's Identities, Green's Functions, Green's Functions for a Half-space and Sphere, Harnack's Inequalities and Theorems.

Course Outcomes: Students are expected to understand topics such as first and second order linear classical PDEs as well as nonlinear equations. Explicate formulae and derive properties of solutions for problems with homogenous and inhomogeneous equations; without boundaries and with boundaries.

Textbooks: Ioannis P Stavroulakis, Stepan A Tersian, Partial Differential Equations: An Introduction with Mathematica and Maple, World Scientific, 2004.

Reference Books:

1. Walter A Strauss, Partial Differential Equations: An introduction, John Wiley & Sons, 2008.
2. Peter J. Olver, Introduction to Partial Differential Equations, Springer, 2014.

ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)
Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35
End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1-1.2	Introduction to partial differential equations, Linear First-order Equations.
2	1.3	The Cauchy Problem for First-order Quasi-linear Equations. Existence and blowup of solution.
3	1.4	Quasi-linear Equations: theory and methods of general solution.
4	Handouts	Classification of system of partial differential equations. Method of solutions for system of partial differential equations.
5	1.5	Fully-nonlinear First-order Equations: Theory and methods of solution.
6	2.1, 2.2	Methods of solution for Linear Equations. Classification and Canonical Forms of Equations in two Independent Variables.
7	3.1, 3.2	The Wave Equation on the Whole Line. D'Alembert Solution, The Wave Equation on the Half-line.
8	3.3	Reflection Method, Mixed Problem for the Wave Equation.
9	**Mid Semester Exam**	
10	3.4	Inhomogeneous Wave Equation.
11	3.5	Conservation of the Energy.
12	4.1	Maximum-minimum Principle for the Diffusion Equation
13	4.2	The Diffusion Equation on the Whole Line.
14	4.3, 4.4	Diffusion on the Half-line. Inhomogeneous Diffusion Equation on the Whole Line.

15	5.1,5.2	Weak Derivatives and Weak Solutions, Conservation Laws.
16	5.3,5.4	Burgers’ Equation, Weak Solutions. Riemann Problem.
17		Review
18	**End Semester Exam**	